Diquarks, Pentaquarks and Dibaryons

Shi-Lin Zhu*
Department of Physics, Peking University, BEIJING 100871, CHINA
(Dated: February 1, 2008)

We explore the connection between pentaquarks and dibaryons composed of three diquarks in the framework of the diquark model. With the available experimental data on H dibaryon, we estimate the Pauli blocking and annihilation effects and constrain the $P = -$ pentaquark $SU(3)_F$ singlet mass. Using the Θ^+ pentaquark mass, we estimate $P = -$ dibaryon mass.

PACS numbers: 12.39.Mk, 12.39.-x
Keywords: Pentaquark, Dibaryon, Diquark

I. INTRODUCTION

Baryons in the conventional quark model are color singlets composed of three quarks. So their color wave function is anti-symmetric. Pauli principle requires the total wave function of three quarks be anti-symmetric. For the $L = 0$ ground state baryons, their orbital wave function is symmetric. Therefore their spin-flavor wave function is totally symmetric, corresponding to the nucleon octet and Delta decuplet with positive parity. The mass splitting between the members of the $SU(6)$ multiplet is caused by either the color-spin interaction from the gluon exchange or the flavor-spin interaction from the pseudoscalar meson exchange.

Quark model has been very successful in the classification of baryons [1]. However, quantum chromodynamics (QCD) as the underlying theory of strong interaction, allows a much richer baryon spectrum. Especially there may exist hybrid baryons (qqqG) and multiquark baryons such as pentaquarks (qqqq$\bar{q}$), dibaryons (qqqqqq) etc. Since Jaffe proposed the H dibaryon in 1977 [2], there has been extensive experimental search of this state. There also exist discussions of other possible dibaryons in literature [3, 4]. Up to now, none of these non-conventional baryon states has been established experimentally except pentaquarks.

The surprising discovery of Θ^+ pentaquark [5, 6] last year is one of the most important events in hadron physics for the past decades. There have appeared more than two hundred pentaquark papers in literature within one year. Its quantum number, internal structure, decay mechanism and underlying dynamics are under heated debate [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

Jaffe and Wilczek proposed the diquark picture for pentaquarks [8]. The diquark is very similar to an anti-quark in many aspects. This feature leads to deep connection between pentaquarks and dibaryons which are composed of three diquarks. In this short note, we will explore this connection.

II. $P = +$ PENTAQUARKS *VS* $P = -$ DIBARYONS

Within the framework of the diquark model, we discuss the connection between $P = +$ pentaquarks and those $P = -$ dibaryons which are composed of three diquarks and contain one orbital excitation between diquarks.

Jaffe and Wilczek proposed that the Θ^+ pentaquark is composed of two diquarks and one strange anti-quark [8]. They argued that the light quarks are strongly correlated. Two light quarks tend to form a scalar diquark in the $\bar{3}_c, \bar{3}_F$ representation whenever possible. The lighter the quark mass, the stronger the correlation. The one-gluon-exchange interaction and the instanton induced interaction seem to support such an idea.

Since the pentaquark is a color singlet, the color wave function of the two diquarks within the pentaquark must be antisymmetric $\mathbf{3_C}$. In order to get an exotic anti-decuplet, the two scalar diquarks combine into the symmetric SU(3) $\bar{\mathbf{6}}_\mathbf{F}$: $[ud]^2$, $[ud][ds]_+$, $[su]^2$, $[su][ds]_+$, $[ds]^2$, and $[ds][ud]_+$. Bose statistics demands symmetric total wave function of the diquark-diquark system, which leads to the antisymmetric spatial wave function with one orbital excitation. The resulting anti-decuplet and octet pentaquarks have $J^P = \frac{1}{2}^+, \frac{3}{2}^+$. The resulting flavor wave functions are collected in Table I.

Throughout our discussion we assume exact isospin symmetry. We denote the up and strange quark mass by m_u, m_s and the [ud], [us] diquark mass by $m_{[ud]}, m_{[us]}$. Since the same quark exists in the two diquarks, Pauli blocking effect may raise the spectrum by $E_{pb}^{L=1}$. However the centrifugal barrier from the orbital excitation makes two diquarks far apart. One expects that the Pauli blocking effect is less significant for $P = +$ pentaquarks than for $P = -$ pentaquarks.

In contrast, the quark and anti-quark annihilation effect tends to lower the spectrum. There are two kinds of possible annihilation mechanism. For example, the $\bar{u}$ may annihilate with the up quark in either [ud] or [us] diquark. Such a mechanism lowers the pentaquark mass by E_{ann}. The second possibility is that the $\bar{u}$ and down quark in the [ud] diquark annihilates into a virtual K or K^*, which may also lower the pentaquark mass by E'_{ann}. E_{ann} is probably greater than E'_{ann}. After taking into

*Electronic address: zhusl@th.phy.pku.edu.cn

account of the Pauli blocking and annihilation effects, Θ^+ and Ξ^{--} masses are

$$M_{\Theta^+} = 2m_{[ud]} + m_s + 2E_{pb}^{L=1} - 4E'_{ann} + E_L \quad (1)$$

$$M_{\Xi^{--}} = 2m_{[us]} + m_u + 2E_{pb}^{L=1} - 4E'_{ann} + E_L \quad (2)$$

We list $P = +$ pentaquark masses in Table II.

Θ^+ pentaquark is interpreted as a bound state of two diquarks and one anti-quark by Jaffe and Wilczek [8]. Its mass is as low as 1530 MeV even with one orbital excitation. One may wonder whether one can get a low lying dibaryon with $L = 1$ after replacing the anti-quark in Θ^+ by a diquark.

Now let's discuss $P = -$ dibaryons composed of three scalar diquarks with $L = 1$. Its color wave function is anti-symmetric. Its spin wave function is symmetric since diquarks are scalars. Bose statistics requires the total wave function is symmetric. Hence the product of the flavor and orbital wave function is anti-symmetric. Suppose there is one orbital excitation between two diquarks: A and B. The flavor wave function of the diquark pair A and B must be symmetric, which is the same as in the $P = +$ pentaquarks. When the orbital wave function is mixed symmetric (or anti-symmteric), the flavor wave function must be mixed anti-symmteric (or symmetric). This situation is very similar to the $L = 1$ baryon multiplet in the $SU(6)_{FS}$ 70_{FS} representation. The only difference is that the diquark is a scalar. Simple group theory tells us that the resulting $P = -$ dibaryons are in the 8_F representation.

To some extent one may correspond [ud], [us], [ds] diquarks to $\bar{S}, \bar{D}, \bar{U}$ respectively. We classify the dibaryon type depending on its $\bar{S}, \bar{D}, \bar{U}$ content. For example, the quark content of the proton-type dibaryon is $\bar{U}\bar{U}\bar{D}$ or $[ds][ds][us]$. We use the lower index "6" to denote the dibaryon. For the Λ-type (or Σ^0-type) dibaryon Λ_6 (or Σ_6^0) with the quark content $[ud][us][ds]$, its mass can be estimated as

$$M_{\Lambda_6,\Sigma_6^0} = 2m_{[us]} + m_{[ud]} + E_L + 2E_{pb}^{L=0} + E_{pb}^{L=1} \quad (3)$$

For the Ξ-type dibaryon Ξ_6 with the quark content $[ud][ud][us]$,

$$M_{\Xi_6} = 2m_{[ud]} + m_{[us]} + E_L + 2E_{pb}^{L=0} + 2E_{pb}^{L=1} \quad (4)$$

For the nucleon-type dibaryon N_6 with the quark content $[us][us][ds]$,

$$M_{N_6} = 3m_{[us]} + E_L + 2E_{pb}^{L=0} + 2E_{pb}^{L=1} \quad (5)$$

For the $\Sigma^{\pm}$-type dibaryon $\Sigma_6^{\pm}$, its mass can be estimated as

$$M_{\Sigma_6^{\pm}} = 2m_{[us]} + m_{[ud]} + E_L + 2E_{pb}^{L=0} + 2E_{pb}^{L=1} \quad (6)$$

III. $P = -$ PENTAQUARKS *VS* $P = +$ DIBARYONS

Let's move on to those dibaryons which are composed of three diquarks and have no orbital excitation. Three $\bar{3}_c$ diquarks combine into a color singlet so their color wave function is antisymmetric. Diquarks are scalars. They obey Bose statistics. Their total wave function should be symmetric. Since there is no orbital excitation between scalar diquarks, their spin and spatial wave functions are symmetric. Hence their flavor wave function must be totally anti-symmetric. I.e., the resulting dibaryon is a $SU(3)_F$ singlet with positive parity, which is nothing but the H dibaryon proposed by Jaffe long time ago [2]. Another P=+ dibaryon with two P-waves between the diquarks and $L = 0$ could also be low lying [23].

Within the diquark framework, it was pointed out that lighter pentaquarks can be formed if the two scalar diquarks are in the antisymmetric $SU(3)_F$ $\mathbf{3}$ representation [20, 24]: $[ud][su]_-$, $[ud][ds]_-$, and $[su][ds]_-$, where $[q_1q_2][q_3q_4]_- = \sqrt{\frac{1}{2}}([q_1q_2][q_3q_4] - [q_3q_4][q_1q_2])$. No orbital excitation is needed to ensure the symmetric total wave function of two diquarks since the spin-flavor-color part is symmetric. The total angular momentum of these pentaquarks is $\frac{1}{2}$ and the parity is negative. There is no accompanying $J = \frac{3}{2}$ multiplet. The two diquarks combine with the antiquark to form a $SU(3)_F$ octet and singlet pentaquark multiplet: $\bar{3}_F \otimes 3_F = \mathbf{8_F} \oplus \mathbf{1_F}$.

We want to emphasize that the above pentaquark singlet with negative parity is very similar to the H dibaryon. Its flavor wave function reads

$$\frac{1}{\sqrt{3}}\left([ud][su]_-\bar{u} + [ds][ud]_-\bar{d} + [su][ds]_-\bar{s}\right) \quad (7)$$

Since the same quark exists within two diquarks, Pauli blocking effect may raise the spectrum by $E_{pb}^{L=0}$. In contrast, the quark and anti-quark annihilation effect tends to lower the spectrum by E_{ann}. Since there is no orbital excitation, the diquarks are in S-wave. $E_{pb}^{L=0}$ can be quite significant and $E_{pb}^{L=0} >> E_{pb}^{L=1}$.

The $P = -$ pentaquark singlet mass may be estimated as

$$\begin{aligned} M_1 = \ & \tfrac{1}{3}\left(2m_u + m_s + 2m_{[ud]} + 4m_{[us]}\right) \\ & + E_{pb}^{L=0} - 2E_{ann} - 2E'_{ann} \end{aligned} \quad (8)$$

Replacing the antiquark in Eq. (7) by the corresponding diquark we arrive at the H dibaryon with the diquark content $[ud][us][ds]$. Its mass reads

$$M_H = m_{[ud]} + 2m_{[us]} + 3E_{pb}^{L=0} \quad (9)$$

IV. DISCUSSION

We follow Ref. [8] and use $m_u = 360$ MeV, $m_s = 460$ MeV, $m_{[ud]} = 420$ MeV, and $m_{[us]} = 580$ MeV. If we

naively ignore the Pauli blocking and annihilation effects, we get

$$M_{\Lambda_6} = M_{\Theta^+} + 2m_{[us]} - m_{[ud]} - m_s = 1710\text{MeV} \quad (10)$$

where we have used $M_{\Theta^+} = 1530$ MeV [5]. Such a low lying dibaryon with negative parity is clearly in conflict with the experimental data. In other words, the Pauli blocking and annihilation effects are important.

We may make a rough estimate of $E_{pb}^{L=0}$ from available experimental information on H dibaryon. If H particle really exists, it must be a very loosely bound state which is close to the $\Lambda\Lambda$ threshold. Its binding energy must be less than a few MeV according to the recent doubly Λ hyper-nuclei experiments [26, 27]. In fact, the lower bound of H dibaryon mass was pushed to be $M_H \geq 2224$ MeV. Then we get

$$E_{pb}^{L=0} \approx 215\text{MeV} \quad (11)$$

It's important to note that $E_{pb}^{L=0}$ is correlated with the diquark mass. We may adjust the values of $m_{[ud]}, m_{[us]}$ within a reasonable range to get a smaller $E_{pb}^{L=0}$.

On the other hand, both E_{ann} [25] and E'_{ann} [17] may be important numerically. For a rough estimate, we use $E_{ann} \approx (50 \sim 100)$ MeV, $E'_{ann} \approx (10 \sim 30)$ MeV. The orbital excitation energy E_L is typically around 240 MeV. From Eq. (1), we get

$$E_{pb}^{L=1} \approx 2E'_{ann} \approx (20 \sim 60)\text{MeV} \quad (12)$$

The presence of the orbital excitation in Θ pentaquark contributes an additional energy $E_L \approx 240$ MeV to its mass. However the centrifugal barrier from the orbital excitation reduces the Pauli blocking energy from $2E_{pb}^{L=0} \approx 430$ MeV to $2E_{pb}^{L=1} \approx (20 \sim 60)$ MeV. This effect and the annihilation effect $-4E'_{ann}$ work together to make Θ^+ pentaquark a low lying baryon.

The singlet pentaquark mass reads

$$M_1 = 1662 - 2E_{ann} - 2E'_{ann} = (1402 \sim 1542)\text{MeV} \quad (13)$$

Clearly this $P = -$ pentaquark singlet state is very probably low-lying in the framework of diquark model. Possible decay channels were suggested for future experimental searches in [20].

Putting everything together, we get a rough estimate of the P=- dibaryon mass:

$$M_{\Lambda_6} = (2270 \sim 2310)\text{MeV} \quad (14)$$

This P=- isoscalar dibaryon state is probably $(40 \sim 80)$ MeV above $\Lambda\Lambda$, ΞN threshold. So it's unstable against P-wave $\Lambda\Lambda$ and ΞN strong decays. But it's possibly stable against $\Xi N\pi$ or $\Sigma\Lambda\pi$ S-wave strong decays. Its width is expected to be not very broad. This state could be searched at RHIC.

The author thanks F. E. Close and Q. Zhao for helpful communications. This project was supported by the National Natural Science Foundation of China under Grant 10375003, Ministry of Education of China, FANEDD and SRF for ROCS, SEM.

[1] Particle Dada Group, Phys. Rev. D 66, 010001 (2002).
[2] R. Jaffe, Phys. Rev. Lett. 38, 195 (1977).
[3] J. L. Ping et al. Phys. Rev. C 62, 054007 (2000).
[4] Z. Y. Zhang et al., Phys. Rev. C 61, 065204 (2000); Q. B. Li et al., Nucl. Phys. A 683, 487 (2001); V. B. Kopeliovich, Nucl. Phys. A 639, 75c (1998).
[5] T. Nakano et al., Phys. Rev. Lett. 91, 012002 (2003).
[6] V. V. Barmin et al., hep-ex/0304040; S. Stepanyan et al., hep-ex/0307018; J. Barth et al., hep-ph/0307083; A. E. Arataynet al., hep-ex/0309042; V. Kubarovsky et al., hep-ex/0311046; A. Airapetian et al., hep-ex/0312044; A. Aleev et al., hep-ex/0401024; COSY-TOF Collaboration, hep-ex/0403011.
[7] D. Diakonov et al., Z. Phys. A 359, 305 (1997).
[8] R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003); hep-ph/0312369.
[9] Shi-Lin Zhu, Phys. Rev. Lett. 91, 232002 (2003).
[10] F. Stancu and D. O. Riska, hep-ph/0307010; A. Hosaka, hep-ph/0307232; C. E. Carlson et al, hep-ph/0312325.
[11] M. Karliner and H. J. Lipkin, hep-ph/0307243.
[12] F. Csikor et al., hep-lat/0309090; S. Sasaki, hep-lat/0310014;T.-W. Chiu, T.-H. Hsieh, hep-ph/0403020.
[13] R. Bijker et al., hep-ph/0310281.
[14] J. Ellis et al., hep-ph/0410127.
[15] T. D. Cohen, hep-ph/0309111, hep-ph/0312191.
[16] X.-C. Song and Shi-Lin Zhu, hep-ph/0403093; Y.-X. Liu, J.-S. Li, C.-G. Bao, hep-ph/0401197.
[17] F. Huang et al., hep-ph/0310040; B. Wu and B.-Q. Ma, hep-ph/0311331
[18] X.-G. He and X.-Q. Li, hep-ph/0403191; H. Y. Cheng et al., hep-ph/0403232.
[19] P.-Z. Huang et al, hep-ph/0311108, hep-ph/0401191; Y.R. Liu et al., hep-ph/0312074, hep-ph/0404123; W. W. Li et al., hep-ph/0312362.
[20] A. Zhang et al., hep-ph/0403210.
[21] A.W. Thomas, K. Hicks and A. Hosaka, hep-ph/0312083; Y. Oh, H. Kim, S. H. Lee, hep-ph/0312229; S. I. Nam, A. Hosaka and H.-C. Kim, hep-ph/0401074; Q. Zhao, J. S. Al-Khalili, hep-ph/0312348.
[22] F. E. Close and J. J. Dudek, Phys.Lett. B586, 75 (2004).
[23] F. E. Close and J. J. Dudek, Phys.Lett. B583, 278 (2004); F. E. Close, private communication.
[24] T. D. Cohen, Talk at the conference of "*Pentaquark States: Structure and Properties*", Trento, Italy, Feb. 2004.
[25] P. Bicudo, hep-ph/0405086.
[26] J. K. Ahn et al., Phys. Rev. Lett. 87, 132504 (2001).

[27] H. Takahashi et al., Phys. Rev. Lett. 87, 212502 (2001).

(Y,I,I_3)	$\overline{\mathbf{10}}$	(Y,I,I_3)	$\mathbf{8}$
(2,0,0)	$[ud]^2\bar{s}$	–	–
(1,$\frac{1}{2}$,$\frac{1}{2}$)	$\sqrt{\frac{2}{3}}[ud][us]_+\bar{s}+\sqrt{\frac{1}{3}}[ud]^2\bar{d}$	(1,$\frac{1}{2}$,$\frac{1}{2}$)	$\sqrt{\frac{1}{3}}[ud][us]_+\bar{s}-\sqrt{\frac{2}{3}}[ud]^2\bar{d}$
(1,$\frac{1}{2}$,-$\frac{1}{2}$)	$\sqrt{\frac{2}{3}}[ud][ds]_+\bar{s}+\sqrt{\frac{1}{3}}[ud]^2\bar{u}$	(1,$\frac{1}{2}$,$\frac{1}{2}$)	$\sqrt{\frac{1}{3}}[ud][ds]_+\bar{s}-\sqrt{\frac{2}{3}}[ud]^2\bar{u}$
(0,1,1)	$\sqrt{\frac{2}{3}}[ud][us]_+\bar{d}+\sqrt{\frac{1}{3}}[us]^2\bar{s}$	(0,1,1)	$\sqrt{\frac{1}{3}}[ud][us]_+\bar{d}-\sqrt{\frac{2}{3}}[us]^2\bar{s}$
(0,1,0)	$\sqrt{\frac{1}{3}}([ud][ds]_+\bar{d}+[ud][us]_+\bar{u}$	(0,1,0)	$\sqrt{\frac{1}{6}}([ud][ds]_+\bar{d}+[ud][us]_+\bar{u})$
	$+[us][ds]_+\bar{s})$		$-\sqrt{\frac{2}{3}}[us][ds]_+\bar{s}$
(0,1,-1)	$\sqrt{\frac{2}{3}}[ud][ds]_+\bar{u}+\sqrt{\frac{1}{3}}[ds]^2\bar{s}$	(0,1,-1)	$\sqrt{\frac{1}{3}}[ud][ds]_+\bar{u}-\sqrt{\frac{2}{3}}[ds]^2\bar{s}$
(-1,$\frac{3}{2}$,$\frac{3}{2}$)	$[us]^2\bar{d}$	–	–
(-1,$\frac{3}{2}$,$\frac{1}{2}$)	$\sqrt{\frac{2}{3}}[us][ds]_+\bar{d}+\sqrt{\frac{1}{3}}[us]^2\bar{u}$	(-1,$\frac{1}{2}$,$\frac{1}{2}$)	$\sqrt{\frac{1}{3}}[us][ds]_+\bar{d}-\sqrt{\frac{2}{3}}[us]^2\bar{u}$
(-1,$\frac{3}{2}$,-$\frac{1}{2}$	$\sqrt{\frac{2}{3}}[ds][us]_+\bar{u}+\sqrt{\frac{1}{3}}[ds]^2\bar{d}$	(-1,$\frac{1}{2}$,-$\frac{1}{2}$)	$\sqrt{\frac{1}{3}}[ds][us]_+\bar{u}-\sqrt{\frac{2}{3}}[ds]^2\bar{d}$
(-1,$\frac{3}{2}$,-$\frac{3}{2}$)	$[ds]^2\bar{u}$	–	–
–	–	(0,0,0)	$\sqrt{\frac{1}{2}}([ud][ds]_+\bar{d}-[ud][us]_+\bar{u})$

TABLE I: Flavor wave functions in Jaffe and Wilczek's model [8]. $[q_1q_2][q_3q_4]_+=\sqrt{\frac{1}{2}}([q_1q_2][q_3q_4]+[q_3q_4][q_1q_2])$ or $[q_1q_2]^2=[q_1q_2][q_1q_2]$ is the diquark-diquark part.

(Y,I,I_3)	$\overline{\mathbf{10}}$	(Y,I,I_3)	$\mathbf{8}$
(2,0,0)	$2m_{[ud]}+m_s+2E_{pb}^{L=1}-4E'_{ann}+E_L$	–	–
(1,$\frac{1}{2}$,$\pm\frac{1}{2}$)	$\frac{1}{3}(4m_{[ud]}+2m_{[us]}+2m_s+m_u$	(1,$\frac{1}{2}$,$\pm\frac{1}{2}$)	$\frac{1}{3}(5m_{[ud]}+m_{[us]}+m_s+2m_u$
	$+4E_{pb}^{L=1}-4E_{ann}-8E'_{ann})+E_L$		$+5E_{pb}^{L=1}-5E_{ann}-7E'_{ann})+E_L$
(0,1,$\pm$1)	$\frac{1}{3}(2m_{[ud]}+4m_{[us]}+m_s+2m_u$	(0,1,$\pm$1)	$\frac{1}{3}(m_{[ud]}+5m_{[us]}+2m_s+m_u$
	$+4E_{pb}^{L=1}-4E_{ann}-8E'_{ann})+E_L$		$+4E_{pb}^{L=1}-5E_{ann}-7E'_{ann})+E_L$
(0,1,0)	$\frac{1}{3}(2m_{[ud]}+4m_{[us]}+m_s+2m_u$	(0,1,0)	$\frac{1}{3}(m_{[ud]}+5m_{[us]}+2m_s+m_u$
	$+3E_{pb}^{L=1}-6E_{ann}-6E'_{ann})+E_L$		$+3E_{pb}^{L=1}-6E_{ann}-6E'_{ann})+E_L$
(-1,$\frac{3}{2}$,$\pm\frac{3}{2}$)	$2m_{[us]}+m_u+2E_{pb}^{L=1}-4E'_{ann}+E_L$	–	–
(-1,$\frac{3}{2}$,$\pm\frac{1}{2}$)	$2m_{[us]}+m_u$	(-1,$\frac{1}{2}$,$\pm\frac{1}{2}$)	$2m_{[us]}+m_u$
	$+\frac{1}{3}(4E_{pb}^{L=1}-4E_{ann}-8E'_{ann})+E_L$		$+\frac{1}{3}(5E_{pb}^{L=1}-5E_{ann}-7E'_{ann})+E_L$
–	–	(0,0,0)	$m_{[ud]}+m_{[us]}+m_u+E_{pb}^{L=1}-2E_{ann}-2E'_{ann}+E_L$

TABLE II: $P=+$ pentaquark masses with the correction from Pauli blocking and the annihilation effects. E_L is the orbital excitation energy.